Kurven Aufgabe 52

$\mathbf{f_{(x)} = (1/6)(1 + x)^3(3 - x)}$

$\mathbf{f_{(x)} = (1/6)(1 + 3x + 3x^2 + x^3)(3 - x)}$

$\mathbf{f_{(x)} = (1/6)(3 + 9x + 9x2 + 3x^3 - x - 3x^2 - 3x^3 - x^4)}$

$\mathbf{f_{(x)} = (1/6)(-x^4 + 6x^2 + 8x + 3)}$

$\mathbf{f_{(x)} = - (1/6)x^4 + x^2 + (4/3)x + 0,5}$

$f'_{(x)} = - (2/3)x^3 + 2x + (4/3)$, $f''_{(x)} = - 2x^2 + 2$, $f'''_{(x)} = - 4x$

Definitionsbereich: $\mathbf{- \infty < x < \infty}$

Wertebereich: $\mathbf{- \infty < f_{(x)} \leq 4,5}$ (siehe Extrempunkte)

Asymptoten: -

Symmetrie: -

Nullstellen:

$- (1/6)x^4 + x^2 + (4/3)x + 0,5 = 0 \mid *(-6)$

$x^4 - 6x^2 - 8x - 3 = 0$

Durch Probieren gefunden $x_1 = -1$

Hornerschema:

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 0 | -6 | -8 | -3 |
| $x_1 = -1$ | | -1 | 1 | 5 | 3 |
| | 1 | -1 | -5 | -3 | 0 |

Polynomdivision:

$$
\begin{array}{l}
\quad x^4 - 6x^2 - 8x - 3 : (x + 1) = x^3 - x^2 - 5x - 3 \\
-(x^4 + x^3) \\
\text{-------------} \\
\qquad\quad -x^3 - 6x^2 - 8x - 3 \\
\qquad -(-x^3 - x^2) \\
\qquad \text{----------------} \\
\qquad\qquad\quad -5x^2 - 8x - 3 \\
\qquad\qquad -(-5x^2 - 5x) \\
\qquad\qquad\quad \text{--------------} \\
\qquad\qquad\qquad\quad - 3x - 3 \\
\qquad\qquad\qquad -(-3x - 3)
\end{array}
$$

--------------
0

Durch Probieren gefunden $x_2 = 3$

| | | | | |
|---|---|---|---|---|
| | 1 | -1 | -5 | -3 |
| $x_2 = 3$ | | 3 | 6 | 3 |
| | 1 | 2 | 1 | 0 |

Polynomdivision:

$x^3 - x^2 - 5x - 3 : (x - 3) = x^2 + 2x + 1$
$-(x^3 - 3x^2)$
------------
$2x^2 - 5x - 3$
$-(2x^2 - 6x)$
---------------
$x - 3$
$-(x - 3)$
---------
0

$x^2 + 2x + 1 = 0$

p, q - Formel:

$p = 2, q = 1$

$x_{3,4} = \frac{-2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 1}$

$x_{3,4} = -1 \pm \sqrt{0}$

$x_{1.3,4} = -1$ (dreifache Nullstelle, Berührpunkt)

**$N_{1,2,3}(-1|0), N_4(3|0)$**

Schnittpunkt mit der y-Achse:

$f_{(0)} = -(1/6) * 0^4 + 0^2 + (4/3) * 0 + 0{,}5 = 0{,}5$

**$S_y(0|0{,}5)$**

Extrempunkte:

$-(2/3)x^3 + 2x + (4/3) = 0 \mid *(-3)$

$2x^3 - 6x - 4 = 0$

Durch Probieren gefunden $x_1 = -1$

Hornerschema:

| | | | | |
|---|---|---|---|---|
| | 2 | 0 | -6 | -4 |
| $x_1 = -1$ | | -2 | 2 | 4 |
| | 2 | -2 | -4 | 0 |

Polynomdivision:

$$
\begin{array}{l}
\quad 2x^3 - 6x - 4 : (x + 1) = 2x^2 - 2x - 4 \\
-(2x^3 + 2x^2) \\
\hline
\qquad -2x^2 - 6x - 4 \\
\quad -(-2x^2 - 2x) \\
\hline
\qquad\qquad -4x - 4 \\
\qquad\quad -(-4x - 4) \\
\hline
\qquad\qquad\quad 0
\end{array}
$$

$2x^2 - 2x - 4 = 0 \; |:2$

$x^2 - x - 2 = 0$

p, q - Formel:

$p = -1, q = -2$

$x_{2,3} = \frac{-(-1)}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-2)}$

$x_{2,3} = 0{,}5 \pm \sqrt{2{,}25}$

$x_{2,3} = 0{,}5 \pm 1{,}5$

$x_2 = 2, f_{(2)} = -(1/6) * 2^4 + 2^2 + (4/3) * 2 + 0{,}5 = 4{,}5$

$x_3 = -1, f_{(-1)} = 0$

$f''_{(-1)} = -2 * (-1)^2 + 2 = 0$

$f''_{(2)} = -2 * 2 + 2 < 0$ --> **Hochpunkt (2|4,5)**

Wendepunkt:

$-2x^2 + 2 = 0 -2$

$-2x^2 = -2 \; |:(-2)$

$x^2 = 1 \; |\sqrt{}$

$x_{1,2} = \pm 1$

$x_1 = 1$, $f_{(1)} = -(1/6) * 1^4 + 1^2 + (4/3) * 1 + 0{,}5 = 2{,}67$

$x_2 = -1$, $f_{(-1)} = 0$

$f'''_{(1)} = -4 * 1 \neq 0$ --> **Wendepunkt (1|2,67)**

$f'''_{(-1)} = -4 * (-1) \neq 0$, $f'_{(-1)} = 0$, $f''_{(-1)} = 0$ --> **Sattelpunkt (-1|0)**

Graph: