Kurven Aufgabe 180

$$\mathbf{f_{(x)} = \frac{3 * \sin x}{2 - \cos x}}$$

x im Bogenmaß

Quotientenregel erste Ableitung:

u = 3 * sin x, u' = 3 * cos x

v = 2 - cosx, v' = - (- sin x) = sin x

$$f'_{(x)} = \frac{3 * \cos x * (2 - \cos x) - \sin x * 3 * \sin x}{(2 - \cos x)^2}$$

$$f'_{(x)} = \frac{6 * \cos x - 3 \cos^2 x - 3 \sin^2 x}{(2 - \cos x)^2} = \frac{6 * \cos x - 3 * (\cos^2 x + \sin^2 x)}{(2 - \cos x)^2}$$

$$f'_{(x)} = \frac{6 * \cos x - 3}{(2 - \cos x)^2}$$

Quotientenregel und Kettenregel zweite Ableitung:

u = 6 * cos x - 3, u' = - 6 * sin x

v = $(2 - \cos x)^2$, v' = 2 * (2 - cos x) * sin x = 2 * sin x * (2 - cos x)

$$f''_{(x)} = \frac{- 6 * \sin x * (2 - \cos x)^2 - 2 * \sin x * (2 - \cos x) * (6 * \cos x - 3)}{(2 - \cos x)^4}$$

$$f''_{(x)} = \frac{(2 - \cos x)*[-12*\sin x + 6*\cos x*\sin x - 12*\sin x*\cos x + 6*\sin x]}{(2 - \cos x)^4}$$

$$f''_{(x)} = \frac{- 6 * \sin x - 6 * \sin x * \cos x}{(2 - \cos x)^3} = \frac{- 6 * \sin x * (1 + \cos x)}{(2 - \cos x)^3}$$

Zur Beurteilung, ob $f'''_{(x)} \neq 0$:

u = - 6 * sin x * (1 + cos x)

Produktregel:

u' = - 6 * cos x * (1 + cos x) - sin x * (- 6 * sin x)

$u' = -6\cos x - 6\cos^2 x + 6\sin^2 x$

$$f'''_{(x)} = \frac{u'}{v} = \frac{-6\cos x - 6\cos^2 x + 6\sin^2 x}{(2-\cos x)^3} \text{ ist } \neq 0 \text{ für alle } x$$

Definitionsbereich: **$0 \leq x \leq 2\pi$**

Wertebereich:

$-1{,}73 \leq f_{(x)} \leq 1{,}73$ (siehe Extrempunkte)

Nullstellen:

$$\frac{3 \cdot \sin x}{2 - \cos x} = 0 \mid \cdot(2 - \cos x)$$

$3 \cdot \sin x = 0 \mid :3$

$\sin x = 0$

$x_1 = 0$ **$N_1\ (0|0)$**

$x_2 = \pi = 3{,}14 \triangleq 180°$ **$N_2\ (3{,}14|0)$**

$x_3 = 2\pi = 6{,}28 \triangleq 360°$ **$N_3\ (6{,}28|0)$**

Schnittpunkt mit der y-Achse:

$$f_{(0)} = \frac{3 \cdot \sin 0}{2 - \cos 0} = 0$$

$S_y\ (0|0)$

Extrempunkte:

$$\frac{6 \cdot \cos x - 3}{(2 - \cos x)^2} = 0 \mid \cdot (2 - \cos x)^2$$

$6 \cdot \cos x - 3 = 0 \mid +3$

$6 \cdot \cos x = 3 \mid :6$

$\cos x = 0{,}5$

$$x_1 = \pi/3 = 1{,}05 \triangleq 60°,\ f_{(1{,}05)} = \frac{3 * \sin 1{,}05}{2 - \cos 1{,}05} = 1{,}73$$

$$f''_{(1{,}05)} = \frac{- 6 * \sin 1{,}05 * (\cos 1{,}05 + 2)}{(2 - \cos 1{,}05)^3} < 0 \text{ --> }$$ **Hochpunkt (1,05|1,73)**

$$x_2 = (5/3)\pi = 5{,}24 \triangleq 300°,\ f_{(5{,}24)} = \frac{3 * \sin 5{,}24}{2 - \cos 5{,}24} = - 1{,}73$$

$$f''_{(5{,}24)} = \frac{- 6 * \sin 5{,}24 * (\cos 5{,}24 + 2)}{(2 - \cos 5{,}24)^3} > 0 \text{ --> }$$ **Tiefpunkt (5,24|-1,73)**

Wendepunkte:

$$\frac{- 6 * \sin x * (\cos x + 1)}{(2 - \cos x)^3} = 0 \mid * (2 - \cos x)^3$$

$- 6 * \sin x * (\cos x + 1) = 0$

$-6 * \sin x = 0 | : -6$

$\sin x = 0$

$x_1 = 0$ **WP_1 (0|0)**

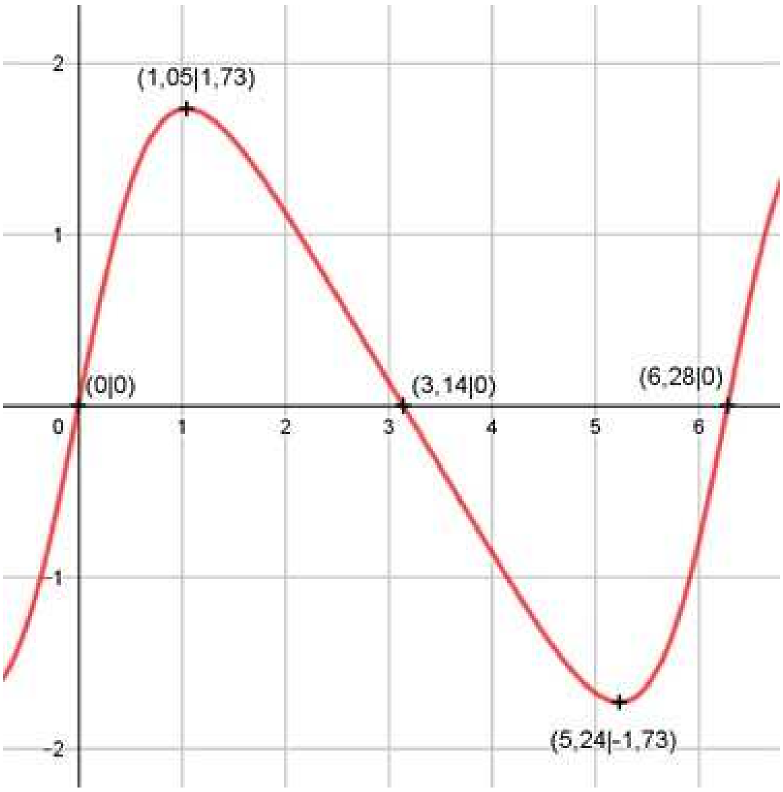
$x_2 = 3{,}14$ **WP_2 (3,14|0)**

$x_3 = 6{,}28$ **WP_3 (6,28|0)**

$\cos x + 1 = 0 \mid -1$

$\cos x = - 1$ --> $x_1 = \pi$

Graph:



(1,05|1,73)
(0|0)
(3,14|0)
(6,28|0)
(5,24|-1,73)
2
1
-1
-2
0
1
2
3
4
5
6