

Integral Aufgabe 139

Berechnen Sie den Flächeninhalt A zwischen $f(x) = x^2 * e^x$ und $g(x) = e^x$ von $x = -1$ bis $x = 1$.

Schnittpunkte:

$$f(x) = g(x)$$

$$x^2 * e^x = e^x \quad | :e^x$$

$$x^2 = 1 \quad | \sqrt{\quad}$$

$x_{1,2} = \pm 1$ entspricht den Grenzen des Intervalls

$$f(x) - g(x) = x^2 * e^x - e^x$$

$$\int (x^2 * e^x) dx$$

Partielle Integration:

$$u = x^2, u' = 2x$$

$$v' = e^x, v = e^x$$

$$\int (x^2 * e^x) dx = x^2 * e^x - \int (2x * e^x) dx$$

Partielle Integration:

$$u = 2x, u' = 2$$

$$v' = e^x, v = e^x$$

$$\int (2x * e^x) dx = 2x * e^x - \int (2 * e^x) dx = 2x * e^x - 2 * e^x$$

$$\int (x^2 * e^x) dx = x^2 * e^x - (2 * x * e^x - 2 * e^x) = e^x * (x^2 - 2x + 2)$$

$$A = \int_{-1}^1 (x^2 * e^x - e^x) dx$$

$$A = \left| e^x(x^2 - 2x + 2) - e^x \right|_{-1}^1 = |e(1 - 2 + 2) - e - [e^{-1}(1 + 2 + 2) - e^{-1}]|$$

$$A = |-1,47|$$

$$\mathbf{A = 1,47}$$

