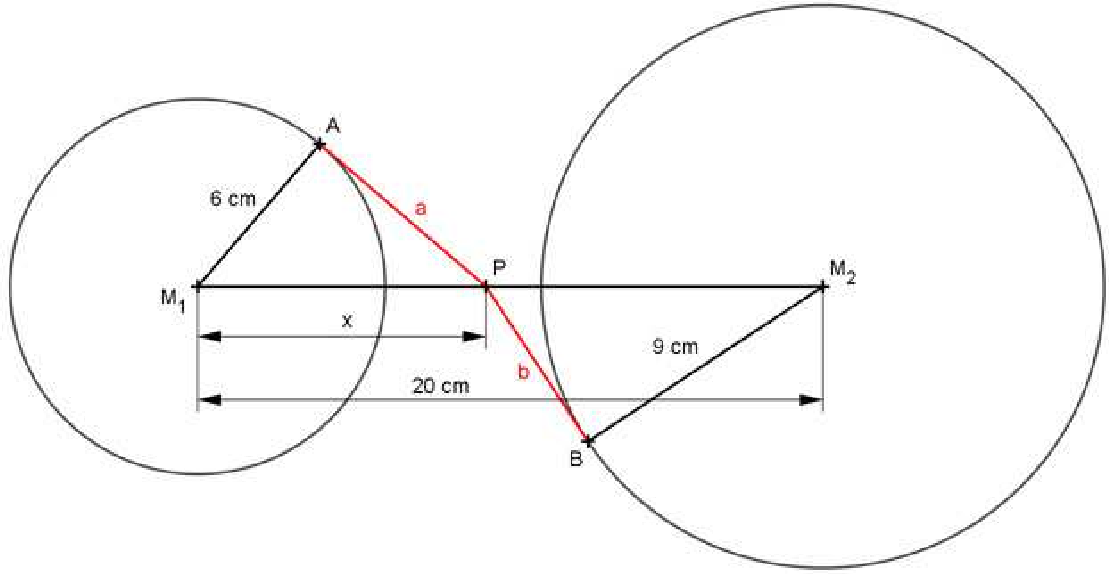
Extrem Aufgabe 69

Wie weit muss P von M_1 entfernt sein, damit die Summe s der beiden Tangenten a und b am größten wird?



Zielfunktion:

$s = a + b$

Nebenbedingung:

Satz von Pythagoras im Dreieck M_1PA:

$x^2 = a^2 + 6^2 \mid -6^2$

$a^2 = x^2 - 36 \mid \sqrt{\ }$

$a = \sqrt{x^2-36}$ $\qquad$ $6 < x < 11$ cm

Satz von Pythagoras im Dreieck PBM_2:

$(20 - x)^2 = 9^2 + b^2 \mid -9^2$

$b^2 = 400 - 40x + x^2 - 81 = x^2 - 80x + 319 \mid \sqrt{\ }$

$b = \sqrt{x^2- 40x + 319}$

In die Zielfunktion eingesetzt:

$s_{(x)} = \sqrt{x^2-36} + \sqrt{x^2- 40x + 319}$

Summen- und Kettenregel:

$$s'_{(x)} = \frac{1}{2} * \frac{2x}{\sqrt{x^2-36}} + \frac{1}{2} * \frac{2x - 40}{\sqrt{x^2- 40x + 319}}$$

$$s'_{(x)} = \frac{x}{\sqrt{x^2-36}} + \frac{x - 20}{\sqrt{x^2- 40x + 319}}$$

$$s'_{(x)} = \frac{x * \sqrt{x^2- 40x + 319} + (x - 20) * \sqrt{x^2-36}}{\sqrt{x^2-36} * \sqrt{x^2- 40x + 319}}$$

$$\frac{x * \sqrt{x^2- 40x + 319} + (x - 20) * \sqrt{x^2-36}}{\sqrt{x^2-36} * \sqrt{x^2- 40x + 319}} = 0 \quad |*\sqrt{x^2-36} * \sqrt{x^2- 40x + 319}$$

$$x * \sqrt{x^2- 40x + 319} + (x - 20) * \sqrt{x^2-36} \quad |- (x - 20) * \sqrt{x^2-36}$$

$$x * \sqrt{x^2- 40x + 319} = - (x - 20) * \sqrt{x^2-36} \quad |^2$$

$$x^2 * (x^2 - 40x + 319) = (x - 20)^2 * (x^2 - 36)$$

$$x^4 - 40x^3 + 319x^2 = (x^2 - 40x + 400) * (x^2 - 36)$$

$$x^4 - 40x^3 + 319x^2 = x^4 - 40x^3 - 364x^2 + 1\,440x - 14\,400 \quad |-x^4 + 40x^3$$

$$319x^2 = 364x^2 + 1\,440x - 14\,400 \quad | - 319\,x^2$$

$$45x^2 + 1\,440x - 14\,400 = 0 \quad | :45$$

$$x^2 + 32x - 320 = 0$$

$$p = 32 \; ; \; q = - 320$$

$$x_{1,2} = \frac{-32}{2} \pm \sqrt{\left(\frac{32}{2}\right)^2 -(-320)}$$

$$x_{1,2} = - 16 \pm 24$$

$\mathbf{x_1 =}$ $- 16 + 24 =$ $\mathbf{8}$

$x_2 = - 16 - 24 = - 40$ keine Lösung, außerhalb des Definitionsbereiches

$a = \sqrt{64 - 36} = 5,3$

$b = \sqrt{64 - 320 + 319} = 7,9$

Zur Beurteilung, ob $s''_{(x)}$ > oder < 0: (Begründung siehe Kurvendiskussion Aufgabe 105)

$u = x * \sqrt{x^2 - 40x + 319} + (x - 20) * \sqrt{x^2 - 36}$

$$u' = 1 * \sqrt{x^2 - 40x + 319} + \frac{1}{2} * \frac{x * (2x - 40)}{\sqrt{x^2 - 40x + 319}} + 1 * \sqrt{x^2 - 36} + (x - 20) * \frac{1}{2} * \frac{2x}{\sqrt{x^2 - 36}}$$

$$u' = \frac{x^2 - 40x + 319 + x^2 - 20x}{\sqrt{x^2 - 40x + 319}} + \frac{x^2 - 36 + x^2 - 20x}{\sqrt{x^2 - 36}}$$

$$s''_{(x)} = \frac{u'}{v} = \frac{\dfrac{x^2 - 40x + 319 + x^2 - 20x}{\sqrt{x^2 - 40x + 319}} + \dfrac{x^2 - 36 + x^2 - 20x}{\sqrt{x^2 - 36}}}{\sqrt{x^2 - 36} * \sqrt{x^2 - 40x + 319}}$$

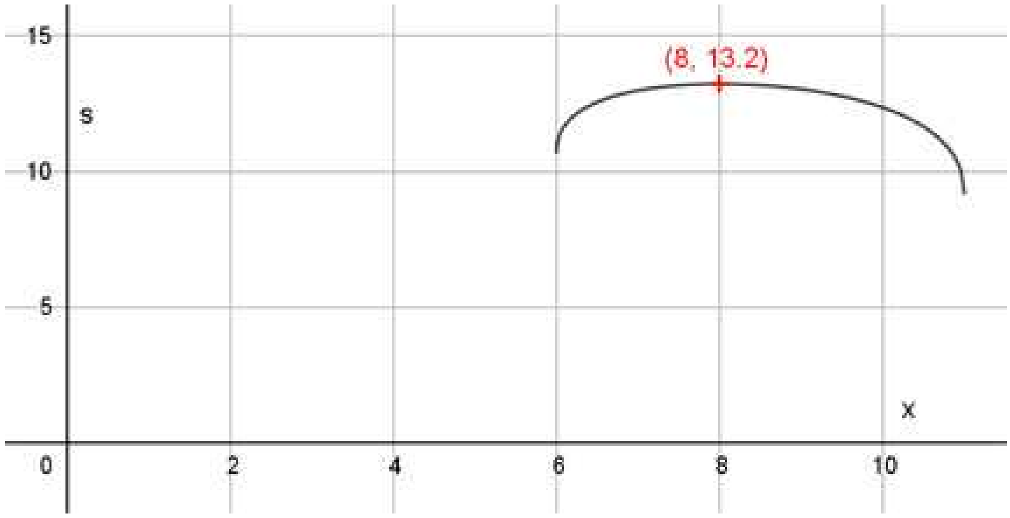
$$s''_{(8)} = \frac{\dfrac{64 - 320 + 319 + 64 - 160}{\sqrt{x^2 - 40x + 319}} + \dfrac{64 - 36 + 64 - 160}{\sqrt{x^2 - 36}}}{\sqrt{x^2 - 36} * \sqrt{x^2 - 40x + 319}}$$

$$s''_{(8)} = \frac{\dfrac{- 33}{7,9} + \dfrac{- 68}{5,3}}{5,3 * 7,9} < 0 \text{ --> Maximum}$$

$s_{(8)} = a + b = 5,3$ cm $+ 7,9$ cm $= 13,2$ cm absolutes Maximum, weil

$s_{(6)} = \sqrt{6^2 - 36} + \sqrt{6^2 - 40 * 6 + 319} = 10,7$ cm $< 13,2$ cm

$s_{(11)} = \sqrt{11^2 - 36} + \sqrt{11^2 - 40 * 11 + 319} = 9,2$ cm $< 13,2$ cm



(8, 13.2)
s
x
15
10
5
0
2
4
6
8
10