

Wurzeln Aufgabe 119

$$\begin{aligned}
 \sqrt{\frac{a}{b} \sqrt{\frac{b}{a}} \sqrt{\frac{a}{b}}} &= \sqrt{\frac{a}{b} \sqrt{\frac{b}{a} \times \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}}} \\
 &= \sqrt{\frac{a \times b^{\frac{1}{2}} \times a^{\frac{1}{2} \times \frac{1}{2}}}{b \times a^{\frac{1}{2}} \times b^{\frac{1}{2} \times \frac{1}{2}}}} = \sqrt{\frac{a \times b^{\frac{1}{2}} \times a^{\frac{1}{4}}}{b \times a^{\frac{1}{2}} \times b^{\frac{1}{4}}}} \\
 &= \frac{a^{\frac{1}{2}} \times b^{\frac{1}{2} \times \frac{1}{2}} \times a^{\frac{1}{4} \times \frac{1}{2}}}{b^{\frac{1}{2}} \times a^{\frac{1}{2} \times \frac{1}{2}} \times b^{\frac{1}{4} \times \frac{1}{2}}} = \frac{a^{\frac{1}{2}} \times b^{\frac{1}{4}} \times a^{\frac{1}{8}}}{b^{\frac{1}{2}} \times a^{\frac{1}{4}} \times b^{\frac{1}{8}}} \\
 &= \frac{a^{\frac{1}{2} + \frac{1}{8} - \frac{1}{4}}}{b^{\frac{1}{2} + \frac{1}{8} - \frac{1}{4}}} = \frac{a^{\frac{4+1-2}{8}}}{b^{\frac{4+1-2}{8}}} \\
 &= \frac{a^{\frac{3}{8}}}{b^{\frac{3}{8}}} = \sqrt[8]{\frac{a^3}{b^3}}
 \end{aligned}$$