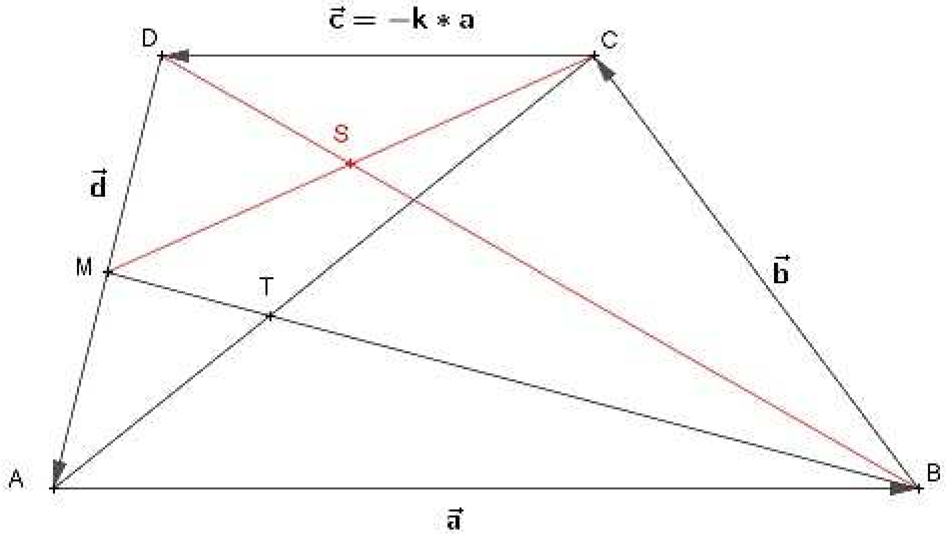
Analytische Geometrie Aufgabe 96

Für das Trapez gilt: $\vec{c} = -k * \vec{a}$. (k > 0). M halbiert AD.
AC schneidet BM im Punkt T.
a) In welchem Verhältnis schneidet T die Strecke AC?
b) In welchem verhältnis schneidet T die Strecke BM?
c) In welchen Verhältnissen schneiden sich BD und CM?
d) Was für eine Bedeutung haben die Fälle, wenn k = 1 oder k = 0?



a)

Geschlossene Vektorkette:

$$\overrightarrow{AT} + \overrightarrow{TB} - \vec{a} = \vec{0}$$

$$\overrightarrow{AT} = \lambda * \overrightarrow{AC} = \lambda * (\vec{a} + \vec{b})$$

$$\overrightarrow{TB} = \mu * \overrightarrow{MB} = \mu * (\frac{\vec{d}}{2} + \vec{a})$$

$$\vec{d} = k * \vec{a} - \vec{b} - \vec{a}$$

$$\overrightarrow{TB} = \mu * (\frac{\vec{a} * (k-1) - \vec{b}}{2} + \vec{a})$$

$$\lambda * \left(\vec{a} + \vec{b}\right) + \mu * \left(\frac{\vec{a} * (1-1) - \vec{b}}{2} + \vec{a}\right) - \vec{a} = \vec{0}$$

$$\vec{a} * \left(\lambda + \mu * \frac{k-1}{2} + \mu - 1\right) + \vec{b} * (\lambda - \frac{\mu}{2}) = \vec{0}$$

$$\lambda - \frac{\mu}{2} + \frac{\mu k}{2} + \mu - 1 = 0$$

$$\lambda + \frac{\mu}{2} + \frac{\mu k}{2} - 1 \quad = 0 \qquad (1)$$

$$\lambda - \frac{\mu}{2} = 0| + \frac{\mu}{2}$$

$$\lambda = \frac{\mu}{2}$$

Eingesetzt in (1):

$$\frac{\mu}{2} + \frac{\mu}{2} + \frac{\mu k}{2} - 1 \quad = 0 \,| * 2$$

$$\frac{\mu}{2} + \frac{\mu}{2} + \frac{\mu k}{2} - 1 \quad = 0 \,| * 2$$

$2\mu + \mu k - 2 = 0 \mid + 2$

$\mu * (k + 2) = 2 \mid : (k + 2)$

$$\mu = \frac{2}{k + 2}$$

$$\lambda = \frac{\frac{2}{k+2}}{2} = \frac{1}{k + 2}$$

$$\frac{\overrightarrow{\mathbf{AT}}}{\overrightarrow{\mathbf{TC}}} = \frac{\lambda * \overrightarrow{AC}}{(1 - \lambda) * \overrightarrow{AC}} = \frac{\frac{1}{k+2}}{1 - \frac{1}{k+2}} = \frac{\frac{1}{k+2}}{\frac{k+2-1}{k+2}} = \frac{\mathbf{1}}{\mathbf{k+1}}$$

b)

$$\frac{\overrightarrow{\mathbf{TB}}}{\overrightarrow{\mathbf{MT}}} = \frac{\mu * \overrightarrow{MB}}{(1 - \mu) * \overrightarrow{MB}} = \frac{\frac{2}{k+2}}{1 - \frac{2}{k+2}} = \frac{\frac{2}{k+2}}{\frac{k+2-2}{k+2}} = \frac{\mathbf{2}}{\mathbf{k}}$$

c)

Geschlossene Vektorkette:

$$\overrightarrow{CS} + \overrightarrow{SD} + k * \vec{a} = \vec{0}$$

$$\overrightarrow{CS} = \lambda * \overrightarrow{CM} = \lambda * (-k * \vec{a} + \frac{\vec{d}}{2})$$

$$\vec{d} = k * \vec{a} - \vec{b} - \vec{a}$$

$$\overrightarrow{CS} = \lambda * (-k * a + \frac{\vec{a} * (k-1) - \vec{b}}{2})$$

$$\overrightarrow{SD} = \mu * \overrightarrow{BD} = \mu * (\vec{b} - k * \vec{a})$$

$$\lambda * \left(-k * a + \frac{\vec{a} * (k-1) - \vec{b}}{2}\right) + \mu * \left(\vec{b} - k * \vec{a}\right) + k * \vec{a} = \vec{0}$$

$$\vec{a} * \left(-k * \lambda + \lambda * \frac{k-1}{2} - \mu * k + k\right) + \vec{b} * (-\frac{\lambda}{2} + \vec{\mu}) = \vec{0}$$

$$-k * \lambda + \lambda * \frac{k-1}{2} - \mu * k + k = 0$$

$$-k * \lambda + \frac{\lambda * k}{2} - \frac{\lambda}{2} - \mu * k + k = 0 \quad (1)$$

$$-\frac{\lambda}{2} + \vec{\mu} = 0 \mid * 2$$

- λ + 2 * μ = 0 |+λ

λ = 2 * μ

Eingesetzt in (1):

$$-2 * k * \mu + \frac{2 * k * \mu}{2} - \frac{2\mu}{2} - \mu * k + k = 0$$

$$-2 * k * \mu - \mu + k = 0 \mid -k \mid$$

-μ * (1 + 2k) = - k |: - (1 + 2k)

$$\mu = \frac{k}{1 + 2k}$$

$$\lambda = \frac{2k}{1 + 2k}$$

$$\frac{\overrightarrow{\mathbf{CS}}}{\overrightarrow{\mathbf{SM}}} = \frac{\lambda * \overrightarrow{CM}}{(1-\ \lambda) * \ \overrightarrow{CM}} = \frac{\frac{2k}{1+2k}}{1-\ \frac{2k}{1+2k}} = \frac{\frac{2k}{k+2}}{\frac{1+2k-2k}{k+2}} = \frac{\mathbf{2k}}{\mathbf{1}}$$

$$\frac{\overrightarrow{\mathbf{SD}}}{\overrightarrow{\mathbf{BS}}} = \frac{\mu * \ \overrightarrow{BD}}{(1-\ \mu) * \ \overrightarrow{BD}} = \frac{\frac{k}{1+2k}}{1-\ \frac{k}{1+2k}} = \frac{\frac{k}{k+2}}{\frac{1+2k-k}{k+2}} = \frac{\mathbf{k}}{\mathbf{1+\ k}}$$

d)

k = 1 --> Aus dem Trapez wird ein Parallelogramm

k = 0 --> Aus dem Trapez wird ein Dreieck.