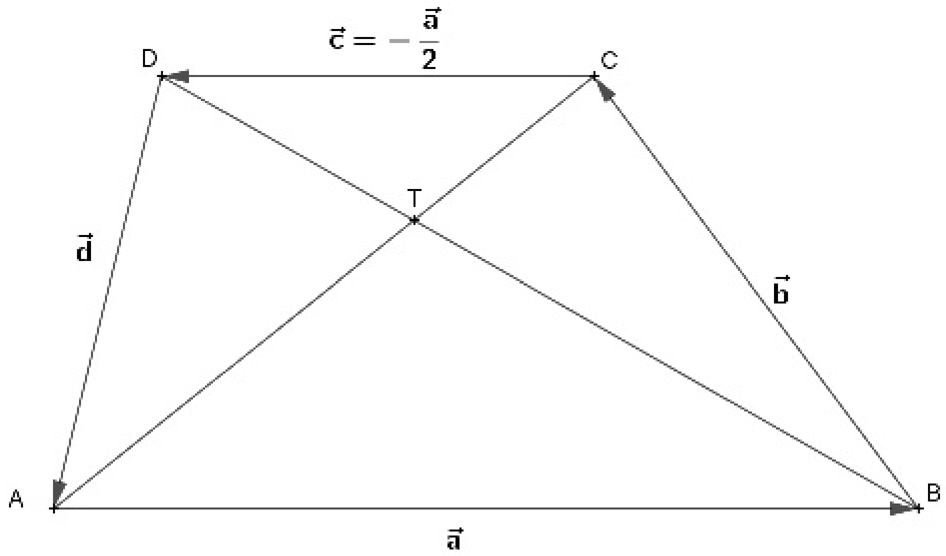
Analytische Geometrie Aufgabe 88

Die Vektoren $\vec{a} = \overrightarrow{AB}, \vec{b} = \overrightarrow{BC}, \vec{c} = -\frac{\vec{a}}{2} = \overrightarrow{CD}$ und $\vec{d} = \overrightarrow{DA}$ bilden das Trapez ABCD.
In welchem Verhältnis teilen sich die Diagonalen?



Geschlossene Vektorkette:

$\overrightarrow{AT} + \overrightarrow{TB} - \vec{a} = \vec{0}$

$\overrightarrow{AT} = \lambda * \overrightarrow{AC}$

$\overrightarrow{AC} = \vec{a} + \vec{b}$

$\overrightarrow{AT} = \lambda * (\vec{a} + \vec{b})$

$\overrightarrow{TB} = \mu * \overrightarrow{DB}$

$\overrightarrow{DB} = -\vec{c} - \vec{b} = -\left(-\frac{\vec{a}}{2}\right) - \vec{b} = \frac{\vec{a}}{2} - \vec{b}$

$\overrightarrow{TB} = \mu * \overrightarrow{DB} = \mu * (\frac{\vec{a}}{2} - \vec{b})$

$\lambda * (\vec{a} + \vec{b}) + \mu * (\frac{\vec{a}}{2} - \vec{b}) - \vec{a} = \vec{0}$

$\vec{a} * \left(\lambda + \frac{1}{2}\mu - 1\right) + \vec{b} * (\lambda - \mu) = \vec{0}$

$\lambda + \frac{1}{2}\mu - 1 = 0$ (1)

$\lambda - \mu = 0$ |+ μ

$\lambda = \mu$

Eingesetzt in (1):

$\mu + \frac{1}{2}\mu - 1 = 0$| +1

1,5μ = 1 |:1,5

$\mu = \frac{2}{3}$

$$\frac{\overrightarrow{\mathbf{TB}}}{\overrightarrow{\mathbf{DT}}} = \frac{\mu * \overrightarrow{DB}}{(1-\mu) * \overrightarrow{DB}} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{\mathbf{2}}{\mathbf{1}}$$