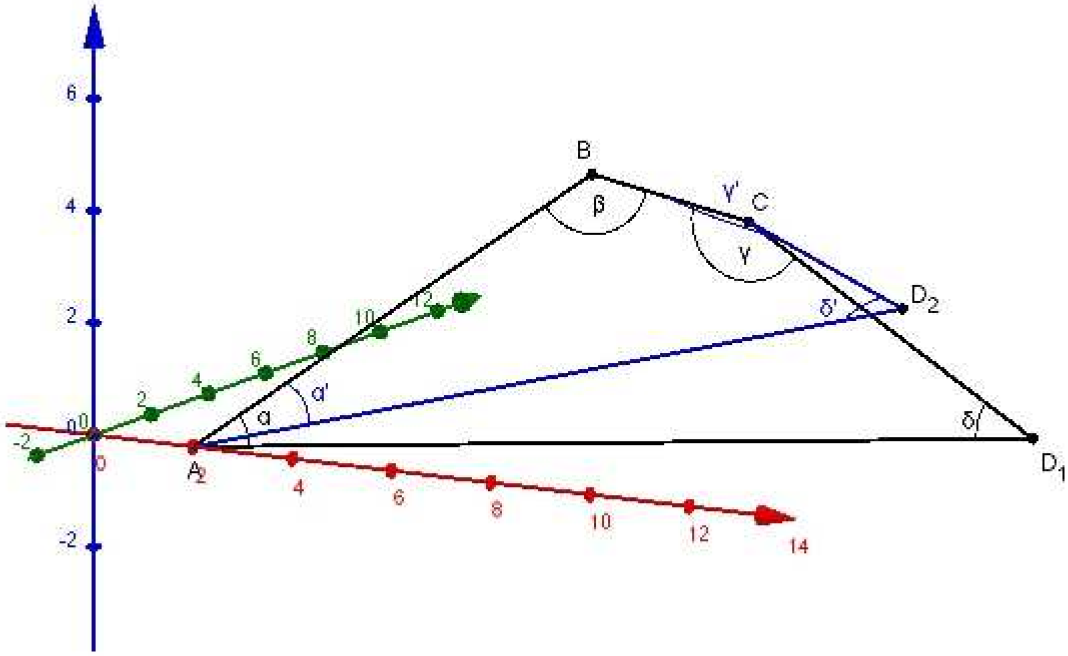
Analytische Geometrie Aufgabe 136

Sind die Vierecke
a) $ABCD_1$ mit A = (2|0|0), B= (6|4|7), C = (8|9|3), D_1 = (12|12|-1)
b) $ABCD_2$ mit D_2 = (14|4|3) eben?



Die Vierecke sind dann eben, wenn ihre Winkelsumme 360° beträgt.

a) Berechnung der Winkel:

$$\cos \alpha = \frac{\overrightarrow{AB} * \overrightarrow{AD_1}}{|\overrightarrow{AB}| * |\overrightarrow{AD_1}|}$$

$$\overrightarrow{AB} = \begin{pmatrix}6\\7\\4\end{pmatrix} - \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}4\\7\\4\end{pmatrix}, \overrightarrow{BA} = \begin{pmatrix}-4\\-7\\-4\end{pmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{4^2 + 7^2 + 4^2} = \sqrt{81} = 9 = |\overrightarrow{BA}|$$

$$\overrightarrow{AD_1} = \begin{pmatrix}12\\12\\-1\end{pmatrix} - \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}10\\12\\-1\end{pmatrix}, \overrightarrow{D_1A} = \begin{pmatrix}-10\\-12\\1\end{pmatrix}$$

$$|\overrightarrow{AD_1}| = \sqrt{10^2 + 12^2 + (-1)^2} = \sqrt{245} = 15{,}65 = |\overrightarrow{D_1A}|$$

$$\cos \alpha = \frac{\begin{pmatrix}4\\7\\4\end{pmatrix} * \begin{pmatrix}10\\12\\-1\end{pmatrix}}{9 * 15{,}65} = \frac{40 + 84 - 4}{140{,}85} = 0{,}852 \dashrightarrow \alpha = 31{,}57°$$

$$\cos \beta = \frac{\overrightarrow{BA} * \overrightarrow{BC}}{|\overrightarrow{BA}| * |\overrightarrow{BC}|}$$

$$\overrightarrow{BA} = \begin{pmatrix}-4\\-7\\-4\end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix}8\\9\\3\end{pmatrix} - \begin{pmatrix}6\\7\\4\end{pmatrix} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$$

$$|\overrightarrow{BA}| = 9$$

$$|\overrightarrow{BC}| = \sqrt{2^2 + 2^2 + (-1)^2} = 3 = |\overrightarrow{CB}|$$

$$\cos \beta = \frac{\begin{pmatrix}-4\\-7\\-4\end{pmatrix} * \begin{pmatrix}2\\2\\-1\end{pmatrix}}{9 * 3} = \frac{-18}{27} = -0{,}6667 \dashrightarrow \beta = 131{,}81°$$

$$\cos \gamma = \frac{\overrightarrow{CB} * \overrightarrow{CD_1}}{|\overrightarrow{CB}| * |\overrightarrow{CD_1}|}$$

$$\overrightarrow{CB} = \begin{pmatrix}-2\\-2\\1\end{pmatrix}, \overrightarrow{CD_1} = \begin{pmatrix}12\\12\\-1\end{pmatrix} - \begin{pmatrix}8\\9\\3\end{pmatrix} = \begin{pmatrix}4\\3\\-4\end{pmatrix}$$

$$|\overrightarrow{CB}| = 3$$

$$|\overrightarrow{CD_1}| = \sqrt{4^2 + 3^2 + (-4)^2} = \sqrt{41} = 6{,}4 = |\overrightarrow{D_1C}|$$

$$\cos \gamma = \frac{\begin{pmatrix}-2\\-2\\1\end{pmatrix} * \begin{pmatrix}4\\3\\-4\end{pmatrix}}{3 * 6{,}4} = \frac{-18}{19{,}2} = -0{,}9375 \dashrightarrow \gamma = 159{,}64°$$

$$\cos \delta = \frac{\overrightarrow{D_1C} * \overrightarrow{D_1A}}{|\overrightarrow{D_1C}| * |\overrightarrow{D_1A}|}$$

$$\overrightarrow{D_1C} = \begin{pmatrix}-4\\-3\\4\end{pmatrix}, \overrightarrow{D_1A} = \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}12\\12\\-1\end{pmatrix} = \begin{pmatrix}-10\\-12\\1\end{pmatrix}$$

$|\overrightarrow{D_1C}| = 6{,}4$

$|\overrightarrow{D_1A}| = 15{,}65$

$$\cos\delta = \frac{\begin{pmatrix}-4\\-3\\4\end{pmatrix} * \begin{pmatrix}-10\\-12\\1\end{pmatrix}}{6{,}4 * 15{,}65} = \frac{80}{100{,}16} = 0{,}7987 \text{ --> } \delta = 37°$$

$$\cos\alpha' = \frac{\overrightarrow{AB} * \overrightarrow{AD_2}}{|\overrightarrow{AB}| * |\overrightarrow{AD_2}|}$$

$$\overrightarrow{AB} = \begin{pmatrix}4\\7\\4\end{pmatrix}$$

$|\overrightarrow{AB}| = 9$

$$\overrightarrow{AD_2} = \begin{pmatrix}14\\4\\3\end{pmatrix} - \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}12\\4\\3\end{pmatrix}, \overrightarrow{D_2A} = \begin{pmatrix}-12\\-4\\3\end{pmatrix}$$

$$|\overrightarrow{AD_2}| = \sqrt{12^2 + 4^2 + 3)^2} = \sqrt{169} = 13 = |\overrightarrow{D_2A}|$$

$$\cos\alpha' = \frac{\begin{pmatrix}4\\7\\4\end{pmatrix} * \begin{pmatrix}12\\4\\3\end{pmatrix}}{9 * 13} = \frac{48 + 28 + 12}{117} = 0{,}7521 \text{ --> } \alpha' = 41{,}23°$$

$$\cos\gamma' = \frac{\overrightarrow{CB} * \overrightarrow{CD_2}}{|\overrightarrow{CB}| * |\overrightarrow{CD_2}|}$$

$$\overrightarrow{CB} = \begin{pmatrix}-2\\-2\\1\end{pmatrix}$$

$|\overrightarrow{CB}| = 3$

$$\overrightarrow{CD_2} = \begin{pmatrix}14\\4\\3\end{pmatrix} - \begin{pmatrix}8\\9\\3\end{pmatrix} = \begin{pmatrix}6\\-5\\0\end{pmatrix}, \overrightarrow{D_2C} = \begin{pmatrix}-6\\5\\0\end{pmatrix}$$

$$|\overrightarrow{CD_2}| = \sqrt{6^2 + (-5)^2} = \sqrt{61} = 7{,}81 = |\overrightarrow{D_2C}|$$

$$\cos \gamma' = \frac{\begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} * \begin{pmatrix} 6 \\ -5 \\ 0 \end{pmatrix}}{3 * 7{,}81} = \frac{-2}{23{,}43} = -0{,}0854 \text{ --> } \gamma' = 94{,}9°$$

$$\cos \delta' = \frac{\overrightarrow{D_2C} * \overrightarrow{D_2A}}{|\overrightarrow{D_2C}| * |\overrightarrow{D_2A}|}$$

$$\overrightarrow{D_2C} = \begin{pmatrix} -6 \\ 5 \\ 0 \end{pmatrix}, \overrightarrow{D_2A} = \begin{pmatrix} -12 \\ -4 \\ 3 \end{pmatrix}$$

$$|\overrightarrow{D_2C}| = 7{,}81$$

$$|\overrightarrow{D_2A}| = 13$$

$$\cos \delta' = \frac{\begin{pmatrix} -6 \\ 5 \\ 0 \end{pmatrix} * \begin{pmatrix} -12 \\ -4 \\ 3 \end{pmatrix}}{13 * 7{,}81} = \frac{52}{101{,}53} = 0{,}5122 \text{ --> } \delta' = 59{,}19°$$

α + β + γ + δ = 31,57° + 131,81° + 159,64° + 37° = 360,02° -->

ebenes Viereck

b)

α' + β + γ' + δ' = 41,23° + 131,81° + 94,9° + 59,19° = 327,13° -->

Viereck ist nicht eben.